1. APPLICATIONS OF MATRICES AND DETERMINANTS

Points to remember :

- Adjoint of a square matrix A =Transpose of the cofactor matrix of A .
- A (adj A) = (adj A) A = $|A|$ I_n.
- $A^{-1} = \frac{1}{|A|}\text{adj } A$.
- (i) $|A^{-1}| = \frac{1}{|A|}$ (ii) $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$ (iii) $(\lambda A)^{-1} = \frac{1}{\lambda}A^{-1}$, Where λ is a non-zero scalar.
- (i) $(AB)^{-1} = B^{-1}A^{-1}$ (ii) $\left(A^{-1}\right)^{-1} = A$
- If A is a non-singular square matrix of order n , then
  (i) $(adjA)^{-1} = adj\left(A^{-1}\right) = \frac{1}{|A|}A$
  (ii) $|adj A| = |A|^{n-1}$
  (iii) $adj(adjA) = |A|^{n-2} A$
  (iv) $adj(\lambda A) = \lambda^{n-1} adj(A)$, λ is a non zero scalar
  (v) $|adj(adjA)| = |A|^{(n-1)^2}$
  (vi) $(adj A)^T = adj\left(A^T\right)$
  (vii) $adj(AB) = (adjB)(adjA)$
- (i) $A^{-1} = \pm\frac{1}{\sqrt{|adj A|}}adj A$ (ii) $A = \pm\frac{1}{\sqrt{|adj A|}}adj(adj A)$
- (i) A matrix A is orthogonal if $AA^T = A^T A = I$
  (ii) A matrix A is orthogonal if and only if A is non-singular and $A^{-1} = A^T$
- Methods to solve the system of linear equations A X = B
  (i) By matrix inversion method $X = A^{-1}B$, $|A| \neq 0$
  (ii) By Cramer's rule $x = \frac{\Delta_1}{\Delta}, y = \frac{\Delta_2}{\Delta}, z = \frac{\Delta_3}{\Delta}, \Delta \neq 0$.
  (iii) By Gaussian elimination method
- (i) If $\rho(A) = \rho([A|B])$= number of unknowns, then the system has unique solution.
  (ii) If $\rho(A) = \rho([A|B])$< number of unknowns, then the system has infinitely many solutions.
  (iii) If $\rho(A) \neq \rho([A|B])$ then the system is inconsistent and has no solution.
- The homogenous system of linear equations AX = O
  (i) has the trivial solution, if $|A| \neq 0$.
  (ii) has a non trivial solution, if $|A| = 0$.

BOOK BACK ONE MARKS

1. If $|adj(adjA)| = |A|^9$, then the order of the square matrix A is

1) 3 **2) 4**
3) 2 4) 5

Solution :

$$|adj(adj A)| = |A|^{(n-1)^2}$$

Given $|adj(adj A)| = |A|^9$

$$|A|^{(n-1)^2} = |A|^9$$

$$(n-1)^2 = 9$$

$$n-1 = 3$$

$$n = 4$$

2. If A is a 3 x 3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then $BB^T =$

1) A 2) B
3) I 4) B^T

Solution :

Given $B = A^{-1}A^T$

$$BB^T = (A^{-1}A^T)(A^{-1}A^T)^T$$

$$= (A^{-1}A^T)(A^T)^T(A^{-1})^T$$

$$= (A^{-1}A^T)A(A^{-1})^T$$

$$= (A^{-1}A)A^T(A^T)^{-1}$$

$$= I.I = I$$

3. If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, B = adj A and C = 3A,

then $\frac{|adj\ B|}{|C|} =$

1) $\frac{1}{3}$ **2) $\frac{1}{9}$**
3) $\frac{1}{4}$ 4) 1

Solution :

$$\frac{|adjB|}{|C|} = \frac{|adj(adjA)|}{|3A|} = \frac{|A|^{(n-1)^2}}{3^2|A|}$$

$$= \frac{|A|}{9|A|} = \frac{1}{9}$$

4. If $A\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$, then A =

1) $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ 2) $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$

3) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ 4) $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$

Solution :

$$AX = B \Rightarrow A = BX^{-1}$$

Where $X = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$

$$X^{-1} = \frac{1}{6}\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

$$A = 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\frac{1}{6}\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

5. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then 9I – A =

1) A^{-1} 2) $\frac{A^{-1}}{2}$
3) $3A^{-1}$ **4) $2A^{-1}$**

Solution :

$$9I - A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$$

$$2A^{-1} = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$$

6. If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then

$|adj\ (AB)| =$

1) –40
2) –80
3) –60
4) –20

Solution :

$$AB = \begin{bmatrix} 2+0 & 8+0 \\ 1+10 & 4+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 8 \\ 11 & 4 \end{bmatrix}$$

$$adj(AB) = \begin{bmatrix} 4 & -8 \\ -11 & 2 \end{bmatrix}$$

$$|adj(AB)| = 8 - 88 = -80$$

7. If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of 3x3 matrix A and $|A| = 4$, then x is

1) 15
2) 12
3) 14
4) 11

Solution :

$$|adjA| = |A|^{n-1}$$

$$\begin{vmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{vmatrix} = 4^2$$

$$-6 + 2x = 16$$

$$x = 11$$

8. If $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the value of a_{23} is

1) 0
2) –2
3) –3
4) –1

Solution :

$$|A| = 3(2-0) - 1(-2-0) - 1(4+2)$$

$$= 6 + 2 - 6$$

$$|A| = 2$$

$$a_{23} = \frac{1}{|A|} \text{cofactors of } a_{32}$$

$$= \frac{1}{2}(-1)\begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix}$$

$$= -\frac{1}{2}(0+2) = -1$$

9. If A,B and C are invertible matrices of some order, then which one of the following is not true?

1) adj A = |A| A^{-1}
2) adj (AB) = (adj A) (adj B)
3) det A^{-1} = $(\det A)^{-1}$
4) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

Solution :

Result : adj(AB) = (adj A)(adj B)

10. If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then $B^{-1} =$

1) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$
2) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$
3) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$
4) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

Solution :

since $(AB)^{-1} = B^{-1}A^{-1}$

$$\begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix} = B^{-1}A^{-1}$$

$$B^{-1}\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$$

$$B^{-1}X = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix} \text{ where } X = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

$$X^{-1} = \frac{1}{1}\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix} X^{-1}$$

$$= \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$$

11. If $A^T A^{-1}$ is symmetric, then $A^2 =$

1) A^{-1} 2) $(A^T)^2$

3) A^T 4) $(A^{-1})^2$

Solution :

A is symmetric then $A = A^T$

$A^T A^{-1}$ is symmetric $\Rightarrow A^T A^{-1} = (A^T A^{-1})^T$

$A^T A^{-1} = (A^{-1})^T (A^T)^T$

$A^T A^{-1} = (A^{-1})^T A$

Pre-multiply by A^T on both sides

$A^T A^T A^{-1} = A^T (A^{-1})^T A$

Post- multiply by A on both sides

$(A^T)^2 A^{-1} A = A^T (A^{-1})^T AA$

$(A^T)^2 I = (A^{-1}A)^T A^2$

$(A^T)^2 = A^2$

12. If A is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $(A^T)^{-1} =$

1) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ 2) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$

3) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ 4) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

Solution :

since $(A^T)^{-1} = (A^{-1})^T$

$$(A^T)^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$$

13. If $A = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$ and $A^T = A^{-1}$, then the value of x is

1) $\frac{-4}{5}$ 2) $\frac{-3}{5}$

3) $\frac{3}{5}$ 4) $\frac{4}{5}$

Solution :

since $A^{-1} = A^T$

$AA^T = A^T A = I$ (orthogonal)

$$\frac{1}{5}\begin{bmatrix} 3 & 4 \\ 5x & 3 \end{bmatrix} \frac{1}{5}\begin{bmatrix} 3 & 5x \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\frac{1}{25}\begin{bmatrix} 25 & 15x+12 \\ 15x+12 & 25x^2+9 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

$$\frac{1}{25}[15x+12] = 0$$

$$[15x+12] = 0$$

$$x = \frac{-4}{5}$$

14. If $A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $AB = I$, then $B =$

1) $\left(\cos^2\frac{\theta}{2}\right)A$ 2) $\left(\cos^2\frac{\theta}{2}\right)A^T$

3) $(\cos^2\theta)I$ 4) $\left(\sin^2\frac{\theta}{2}\right)A$

Solution :

$AB = I$

$B = A^{-1}$

$$B = \frac{1}{1+\tan^2\frac{\theta}{2}}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$B = \frac{1}{\sec^2\frac{\theta}{2}} A^T$$

$$B = \cos^2\frac{\theta}{2} A^T$$

15. If A= $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and A(adjA) = $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then k =

1) 0
2) sin θ
3) cos θ
4) 1

Solution :

We know that $A(adjA) = (adjA)A = |A|I$

$\Rightarrow |A| = k$

$k = \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix}$

$k = \cos^2\theta + \sin^2\theta = 1$

16. If A= $\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that λA^{-1} = A, then λ is

1) 17
2) 14
3) 19
4) 21

Solution :

$\lambda A^{-1} = A$

$\lambda \frac{1}{|A|} adjA = A$

$\lambda \frac{1}{-19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$

$\frac{\lambda}{19}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$

$\frac{\lambda}{19} = 1 \Rightarrow \lambda = 19$

17. If adjA = $\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and adjB = $\begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ then adj(AB) is

1) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$
2) $\begin{bmatrix} \mathbf{-6} & \mathbf{5} \\ \mathbf{-2} & \mathbf{-10} \end{bmatrix}$
3) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$
4) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

Solution :

$adj(AB) = (adjB)(adjA)$

$= \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$

18. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

1) 1
2) 2
3) 4
4) 3

Solution :

$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix} \xrightarrow[R_3 \to R_3 + R_1]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Rank is 1

19. If $x^a y^b = e^m, x^c y^d = e^n$, $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the values of x and y are respectively.

1) $e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$
2) log (Δ_1/Δ_3), log (Δ_2/Δ_3)
3) log (Δ_2/Δ_1), log (Δ_3/Δ_1)
4) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$

Solution :

$x^a y^b = e^m, x^c y^d = e^n$

Taking log on both sides

$a \log x + b \log y = m$

$c \log x + d \log y = n$

$\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \neq 0$

$\log x = \frac{\Delta_1}{\Delta_3}$ $\quad \log y = \frac{\Delta_2}{\Delta_3}$

$x = e^{\frac{\Delta_1}{\Delta_3}}$ $\quad y = e^{\frac{\Delta_2}{\Delta_3}}$

20. Which of the following is/are correct?

(i) Adjoint of a symmetric matrix is also a symmetric matrix.

(ii) Adjoint of a diagonal matrix is also a diagonal matrix.

(iii) If A is a square matrix of order n and λ is a scalar, then $adj(\lambda A) = \lambda^n \, adj\,(A)$

(iv) $A(adj\ A) = (adj\ A)\ A = |A|I$

1) only (i) 2) (ii) and (iii)

3) (iii) and (iv) **4) (i), (ii) and (iv)**

Solution :

Result: $adj(\lambda A) = \lambda^{n-1} adj(A)$ is true.

21. If $\rho(A) = \rho([A|B])$, then the system $AX = B$ of linear equations is

1) consistent and has a unique solution

2) consistent

3) consistent and has infinitely many solution

4) inconsistent

Solution :

Ans (2) consistent.

22. If $0 \le \theta \le \pi$ and the system of equations $x + (\sin\theta)y - (\cos\theta)z = 0$, $(\cos\theta)x - y + z = 0$, $(\sin\theta)x + y - z = 0$ has a non-trivial solution then θ is

1) $\frac{2\pi}{3}$ 2) $\frac{3\pi}{4}$

3) $\frac{5\pi}{6}$ **4) $\frac{\pi}{4}$**

Solution :

$$A = \begin{bmatrix} 1 & \sin\theta & -\cos\theta \\ \cos\theta & -1 & 1 \\ \sin\theta & 1 & -1 \end{bmatrix}$$

The system has non-trivial solution if $|A| = 0$

$$\begin{vmatrix} 1 & \sin\theta & -\cos\theta \\ \cos\theta & -1 & 1 \\ \sin\theta & 1 & -1 \end{vmatrix} = 0$$

$$1(0) - \sin\theta(-\cos\theta - \sin\theta) - \cos\theta(\cos\theta + \sin\theta) = 0$$

$$\sin\theta\cos\theta + \sin^2\theta - \cos^2\theta - \cos\theta\sin\theta = 0$$

$$\sin^2\theta = \cos^2\theta$$

$$\theta = \frac{\pi}{4}$$

23. The augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda-7 & \mu+5 \end{bmatrix}$. The system has infinitely many solutions if

1) $\lambda = 7, \mu \ne -5$ 2) $\lambda = -7, \mu = 5$

3) $\lambda \ne 7, \mu \ne -5$ **4) $\lambda = 7, \mu = -5$**

Solution :

When $\lambda = 7, \mu = -5$

$$[A|B] = \begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\rho(A) = \rho[A|B] = 2 < 3$$

The system is consistent and has infinitely many solutions

24. Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $4B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$.

If B is the inverse of A, then the value of x is

1) 2 2) 4

3) 3 **4) 1**

Solution :

Since B is the inverse of A

AB = I

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \frac{1}{4} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\frac{1}{4} a_{13} = 0$$

$$\frac{1}{4}[-2 - x + 3] = 0$$

$$\frac{1}{4}[1 - x] = 0$$

$$x = 1$$

25. If A = $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then adj(adjA) is 1) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ 2) $\begin{bmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{bmatrix}$ 3) $\begin{bmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{bmatrix}$ 4) $\begin{bmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{bmatrix}$	***Solution :*** $\lvert A \rvert = 3(1) + 3(2) + 4(-2)$ $= 3 + 6 - 8$ $\lvert A \rvert = 1$ $adj(adjA) = \lvert A \rvert^{n-2} A$ $= 1.A = A$

BOOK SUMS ***(Exercise and Examples) :***

1. If A = $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$, verify that A(adj A) = (adj A)A = $|A|I_3$.

2. If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is non-singular, find A^{-1}.

3. Find the inverse of the matrix $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$.

4. If A is a non-singular matrix of odd order, prove that |adj A| is positive.

5. Find a matrix A if adj(A) = $\begin{bmatrix} 7 & 7 & -7 \\ -1 & 11 & 7 \\ 11 & 5 & 7 \end{bmatrix}$.

6. If adj A = $\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}.

7. If A is symmetric, prove that adj A is also symmetric.

8. Verify the property $(A^T)^{-1} = (A^{-1})^T$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$

9. Verify $(AB)^{-1} = B^{-1}A^{-1}$ with $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$

10. If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $A^2 + xA + yI_2 = O_2$. Hence, find A^{-1}.